Conditional expectation given the sum when variables have regularly varying densities

Christian Y. Robert

ISFA, Université Lyon 1
50, avenue Tony Garnier, Lyon
France

Joint work with: Michel Denuit, Patricia Ortega Jimenez

Stochastic monotonicity of two independent random variables X and Y given the value of their sum $S = X + Y$ has been linked to log-concave densities since Efron (1965). However, the log-concavity assumption is not realistic in some applications because it excludes heavy-tailed distributions. This paper considers random variables with regularly varying densities to illustrate how heavy tails can lead to a non-monotonic behavior for the conditional expectation $m_X(s) = \mathrm{E}[X|S = s]$, which turns out to be problematic in signal processing or in risk sharing, for instance. This paper first aims to identify situations where a non-monotonic behavior appears according to the tail-heaviness of X and Y. The analysis is then extended to zero-augmented probability distributions. Consequences for signal processing and risk sharing are discussed. Many numerical examples illustrate the results.